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Detecting quantum signatures of optical fields by ultrasmall Josephson junctions

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Abstract. We prove that a mesoscopic Josephson junction, irradiated with a quantum superposition of two 180°-out of phase optical coherent states, exhibits an experimentally observable sensitivity to the quantum coherences of the field state.

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The superposition principle and the consequent interference of probability amplitudes turn out to be the most distinctive aspects of quantum mechanics.

In this paper we consider a physical system consisting of a dc voltage-biased mesoscopic Josephson junction (JJ) irradiated by a single-mode quantized field of frequency $\omega \geq 2\pi \times 10^{14}$ Hz. We show that exposing the junction to a field prepared into a quantum linear combination of two 180°-out of phase coherent states $|\alpha\rangle$ and $|-\alpha\rangle$ [1,2], then the dc component of the expectation value of the supercurrent operator exhibits an experimentally observable behaviour sensitive to the relative quantum phase between the two coherent components. Our approach clearly puts into evidence that such a behaviour is strongly related to the quantum-mechanical nature of the macroscopic phase difference variable φ across the junction and of the supercharge conjugate variable q. The novel results presented in this paper demonstrate the possibility of detecting quantum signatures of an optical single-mode electromagnetic field through interference effects on the Cooper pairs current. In addition, our theory transparently proves that an ultrasmall JJ coupled to monochromatic nonclassical light may be successfully used to distinguish pure states of radiation against statistical mixtures. Our theoretical analysis demonstrates that ultrasmall JJs provide a very promising device for testing the superposition principle, enriching and enhancing a point of view recently introduced by Vourdas [3–5].

The problem of the non-linear dynamics of a JJ coupled to an electromagnetic field has been investigated both in a classical [6,7] and in a quantum context [8,9] introducing more or less restrictive approximations.

Mesoscopic JJs with very small capacitances $C(\leq 10^{-15}$ F) are today practically realized [10] and operate at very low temperatures $T \ll \hbar\Omega/k_{\mathrm{B}} \approx 1$ K. Their harmonic regime is characterized by the plasma frequency $\Omega = \sqrt{E_{\mathrm{J}}E_{\mathrm{C}}}/\hbar \leq 10^{12}$ s^{-1}. In such conditions, the quantum nature of the JJ cannot be ignored, since the Coulomb coupling constant E_{C} is of the same order of or larger than the Josephson coupling constant $E_{\mathrm{J}}(\approx 10^{-22}$ J), where $CE_{\mathrm{C}} = (2e)^2$ and $2eE_{\mathrm{J}} = \hbar I_{\mathrm{cr}}$, I_{cr} being the critical current of the JJ. It is worth emphasizing that for our purposes we don't work in the Coulomb blockade-like regime characterized by the condition $E_{\mathrm{C}} \gg E_{\mathrm{J}}$. We simply require that $E_{\mathrm{C}} \sim E_{\mathrm{J}}$ in such a way that it is impossible to neglect the quantum nature of the JJ but, at the same time, we can still work in the phase representation.

The occurrence of quantum behaviour for the JJ is taken into account adopting the commutation rule $[\varphi, q] = 2ei$ [11]. Starting from the Hamiltonian of an isolated JJ [12]

$$H_0 = \frac{q^2}{2C} + E_{\mathrm{J}}(1 - \cos\varphi) \tag{1}$$

and using the minimal coupling-like rule

$$q \rightarrow q + C(V_0 + V_{\mathrm{F}}) \tag{2}$$

the junction-field Hamiltonian can be cast in the form:

$$H = \frac{[q + C(V_0 + V_{\mathrm{F}})]^2}{2C} + E_{\mathrm{J}}(1 - \cos\varphi) + \hbar\omega\left(a^{\dagger}a + \frac{1}{2}\right) \tag{3}$$

where V_0 and V_{F} are respectively the classical and nonclassical electromotive forces imposed on the superconductive device. In fact, adopting the Voltage Bias Model [13] in accordance to which: 1) the effect of external radiation is to produce an alternating electric field of the same frequency

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across the junction; 2) both the electric field and the current density are constant all over the sample, then $V_0+V_{\rm F}$ may be thought of as the effective voltage difference across the two electrodes of the mesoscopic junction.

Since the field dynamical variable $V_{\rm F}$ appears explicitly in equation (3), we derive its expression in terms of the usual annihilation and creation operators a and $a^\dagger$ of the monochromatic external field photon. To this end we note that, making the dipole approximation $\exp(\mathrm{i}\mathbf{k}\cdot\mathbf{r})\approx 1$ embodied in the Voltage Bias Model, the following Schrödinger representation of the operator $\mathbf{E}$ in vacuum

$$\mathbf{E}=\mathrm{i}\left(\frac{\hbar\omega}{2\varepsilon_0 V}\right)^{1/2}\hat{\mathbf{u}}\left\{a\exp\left[\mathrm{i}\mathbf{k}\cdot\mathbf{r}\right]-a^\dagger\exp\left[-\mathrm{i}\mathbf{k}\cdot\mathbf{r}\right]\right\} \quad (4)$$

may be approximated as

$$\mathbf{E}=\mathrm{i}\left(\frac{\hbar\omega}{2\varepsilon_0 V}\right)^{1/2}\hat{\mathbf{u}}(a-a^\dagger) \quad (5)$$

where V is the quantization volume and the polarization unit vector $\hat{\mathbf{u}}$ of the field mode, assumed linearly polarized, is taken perpendicular to the plane electrodes of the JJ. Thus the expression of the operator $V_{\rm F}$ to be inserted into the Hamiltonian (3) may be written down as

$$V_{\rm F}=|\mathbf{E}|d=\mathrm{i}\left(\frac{\hbar\omega}{2C_{\rm F}}\right)^{1/2}(a-a^\dagger) \quad (6)$$

where d is the JJ thickness (typically $d\approx 1$ nm), $\varepsilon_{\rm r}$ the relative dielectric constant of the thin insulating barrier ($\varepsilon_{\rm r}\approx 6$) and the capacitive parameter $C_{\rm F}$ is defined as

$$C_{\rm F}=\frac{\varepsilon}{d^2}\left(\frac{\pi c}{\omega}\right)^3. \quad (7)$$

In the Heisenberg picture the representative operators of the field-JJ system evolve as follows:

$$\dot{a}=-\mathrm{i}\omega a-(q+CV_0)\sqrt{\frac{\omega}{2\hbar C_{\rm F}}}-\mathrm{i}\frac{\omega}{2}\frac{C}{C_{\rm F}}(a-a^\dagger) \quad (8)$$

$$\dot{a}^\dagger=\mathrm{i}\omega a^\dagger-(q+CV_0)\sqrt{\frac{\omega}{2\hbar C_{\rm F}}}-\mathrm{i}\frac{\omega}{2}\frac{C}{C_{\rm F}}(a-a^\dagger) \quad (9)$$

$$I\equiv-\dot{q}=I_{\rm cr}\sin\varphi \quad (10)$$

$$\dot{\varphi}=\frac{2e}{\hbar}\left(\frac{q}{C}+V_0+V_{\rm F}\right). \quad (11)$$

To treat this complicated operator coupled differential system we must resort to some approximations. To this end it seems appropriate, at this point, to make clear the importance of keeping the operating temperature well below 1 K. On the one hand, such a condition, in fact, somehow legitimates the expectation that quantum effects in the supercurrent crossing our very low capacitance Josephson junction are not washed out due to thermal fluctuations. On the other hand, when conditions for neglecting this source of dissipative effects are met, charge fluctuations characterizing the dynamics of a JJ working in mesoscopic regime, may be taken of the order of $2e$. In other words, we are assuming that the interaction between the JJ and the dissipative environment is controllable at level such to guarantee the quantumness of the nanodevice.

Coming back to system (8–11), let's choose $\omega\approx 2\pi\times 10^{14}$ Hz so that $C\ll C_{\rm F}\approx 10^{-12}$ F. This fact allows neglecting the terms proportional to $C/C_{\rm F}$ in equations (8, 9) with respect the other terms. Moreover, at the light of the previous digression, due to the very small intensity of the supercurrent crossing the ultrasmall sized junction (order of magnitude of $[q]\sim 2e$), we can neglect all the backreaction corrections on the external field. Thus, in accordance with such arguments, the time evolution of the field mode is given by the equations

$$\dot{a}\approx-\mathrm{i}\omega a \quad (12)$$

$$\dot{a}^\dagger\approx\mathrm{i}\omega a^\dagger. \quad (13)$$

describing a free field evolution.

Finally, since under our assumptions $\omega\gg\Omega\approx 10^{12}$ Hz, H_0, given by equation (1), determines an evolution, characterized by the plasma frequency Ω of the JJ, which is much slower than that controlled by ω and due to the terms describing effects from external field in the Hamiltonian (3). In this way, in the time interval $\omega^{-1}<\Delta t<\Omega^{-1}$, we legitimately neglect the term proportional to q in equation (11) with respect to $V_0+V_{\rm F}$. The operators φ and q, therefore, evolve in accordance with the following approximated equations

$$I\equiv-\dot{q}=I_{\rm cr}\sin\varphi \quad (14)$$

$$\dot{\varphi}\approx\frac{2e}{\hbar}(V_0+V_{\rm F}). \quad (15)$$

It is of relevance to underline that albeit the approach leading from the system (8–11) to the new system (12–15) is very crude, the dynamical behaviour of the JJ as described by equations (14, 15) yet exhibits the quantum nature of the nanodevice.

Integrating equations (12, 13) we derive the dynamical behaviour of $V_{\rm F}$ and successively of the supercurrent $I(t)$ after the explicit integration of equation (15). In this way we definitively get

$$I(t)=I_{\rm cr}\sin\left\{\frac{2e}{\hbar}\left[V_0 t-\sqrt{\frac{\hbar}{2\omega C_{\rm F}}}\times\left[a(0)\exp(-\mathrm{i}\omega t)+a^\dagger(0)\exp(-\mathrm{i}\omega t)\right]\right]\right\}. \quad (16)$$

Let's suppose that the initial density matrix ρ describing the field-JJ system may be factorized as

$$\rho(0)=\rho_{\rm JJ}(0)\otimes\rho_{\rm F}(0) \quad (17)$$

where $\rho_{\rm JJ}(0)(\rho_{\rm F}(0))$ describes the material subsystem (radiation field) at $t=0$.

It is possible to calculate the expectation value of the supercurrent $I(t)$ by taking the trace of the operator $I(t)$ multiplied by the reduced initial field density matrix $\rho_{\rm F}(0)$:

$$\langle I(t)\rangle=\mathrm{Tr}\left[\rho_{\rm F}(0)I(t)\right]. \quad (18)$$

We have in fact demonstrated [14] that contributions coming from tracing out also the phase states of the junction introduced a simple proportionality factor only which does not play any essential role when the interest is mainly focussed on the structure of the $I-V$ characteristic of the JJ.

Suppose the field initially prepared into the state

$$\rho_{\mathrm{F}}(0) = |\Psi\rangle\langle\Psi| \tag{19}$$

with

$$|\Psi\rangle = \frac{1}{G}\left\{|\alpha\rangle + \exp(\mathrm{i}\theta)|-\alpha\rangle\right\} \tag{20}$$

where $G = \left\{2 + 2\cos\theta\exp(-2|\alpha|^2)\right\}^{1/2}$ is the normalization factor and $|\alpha\rangle = ||\alpha|\exp(\mathrm{i}\theta_\alpha)\rangle$.

Quite recently there have been several proposals for the generation of quantum superpositions of two coherent states [15,16]. The interest toward these states, also known as Schrödinger cats, stems from the consideration that, due to probability amplitude interference effects [17], they might exhibit nonclassical θ-dependent properties pronounced enough to provide almost ideal conditions to test basic aspects of quantum mechanics. For example, putting $\theta = 0$ ($\theta = \pi$) we obtain the so called even (odd) coherent states characterized by oscillations of the photon number distribution which are absent in the Yurke and Stoler coherent state ($\theta = \pi/2$) manifesting on the contrary higher order squeezing [17]. It is important to underline that the interference of quantum amplitudes results from the linear superposition principle that is one of the most fundamental features of quantum mechanics. After a lengthy calculation the time- dependent expression of the supercurrent $\langle I(t)\rangle$ can be cast in the following form ($\omega_0 = 2eV_0/\hbar$):

$$\frac{\langle I(t)\rangle}{I_{\mathrm{cr}}} = \frac{\exp(-\xi^2/2)}{G^2}\left[I_{\mathrm{MIXT}} + I_{\mathrm{INT}}\right] \tag{21}$$

with

$$I_{\mathrm{MIXT}} = \sum_{K=-\infty}^{+\infty} J_K(2\xi|\alpha|) \times \sin[(\omega_0 + K\omega)t - K\theta_\alpha]\left(1 + (-1)^K\right) \tag{22a}$$

$$I_{\mathrm{INT}} = \exp(-2|\alpha|^2)\sum_{K=-\infty}^{+\infty} I_K(2\xi|\alpha|)\left\{\sin[(\omega_0 + K\omega)t - K\theta_\alpha + \theta] + (-1)^K\sin[(\omega_0 + K\omega)t - K\theta_\alpha - \theta]\right\} \tag{22b}$$

where $\xi = e\sqrt{2}/\sqrt{\hbar\omega C_{\mathrm{F}}}$ is a real number. The term I_{INT} appearing in the right hand side of equation (21) reflects quantum interference effects between $|\alpha\rangle$ and $|-\alpha\rangle$, related to the specific initial condition of the field described by equation (19).

Equation (21) expresses the main result of this paper: *a Josephson junction operating in a mesoscopic regime provides a sensitive device for detecting quantum coherences present in the state of the radiative field which it is coupled to.*

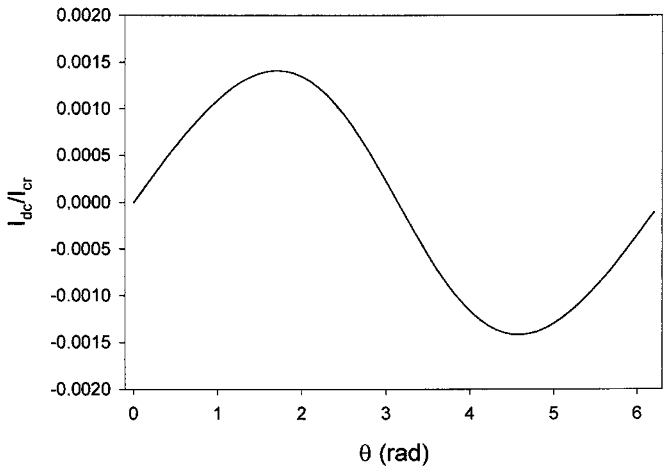

Fig. 1. $I_{\mathrm{dc}}/I_{\mathrm{cr}}$ *versus* θ for the state with $\alpha = |\alpha| = 1$ and $\omega_0/\omega = 1$. Note the different behaviour for $\theta = 0, \pi$ (even and odd coherent states) and $\theta = \pi/2$ (Yurke-Stoler state).

In this sense we may claim that some properties exhibited by the supercurrent mean value, in the context of our theory, provide a direct test of the linear superposition principle. Equations (21, 22) show that, when $\omega_0 = N\omega$, or $2eV_0 = N\hbar\omega$, with N arbitrary integer, the supercurrent would not average to zero. In this case, in fact, the argument of one of the sinusoidal functions appearing in both the summations present in equations (22), does not depend on time and therefore dc current spikes appear. These current spikes may be thought of as the quantum analogue of the well-known Shapiro steps [18].

The key point of our result is that these current singularities, occurring at voltages $V_0 = N(\hbar\omega/2e)$ on the $I-V$ characteristic of the JJ, exhibit details qualitatively and quantitatively dependent on the field state parameters $|\alpha|$, θ_α and θ. This circumstance opens up the possibility of relating the observable structure of the quantum Shapiro steps to the quantum coherences of $|\Psi\rangle$ thusly exploiting the effects of the quantum interference embodied in the expression of $\langle I(t)\rangle$.

Had we started from the statistical mixture

$$\rho_{\mathrm{F}}(0) = \frac{1}{\sqrt{2}}\left[|\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle-\alpha|\right] \tag{23}$$

instead of the pure state (19), we should have obtained only the term I_{MIXT} (with $G^2 = 2$) in equation (21). This means that all the quantum interference effects are embodied in the term I_{INT}. We note that, choosing $\theta_\alpha = 0$, I_{MIXT} cannot give dc contributions whatever the value of the integer K is. Thus no Shapiro step appears in the $I-V$ characteristic of the JJ when the two coherent states $|\alpha\rangle$ and $|-\alpha\rangle$ are only classically superimposed at $t = 0$. A very different behaviour occurs when the initial field state is described by equations (19, 20). In this case in fact, $\alpha \in \Re$ and ω_0 odd multiple of ω, lead to the appearance of Shapiro steps in the $I-V$ characteristic. In Figure 1 we show $I_{\mathrm{dc}}/I_{\mathrm{cr}}$ *versus* θ. We see that, at least in principle, the JJ device is suitable for distinguishing different quantum coherences between $|\alpha\rangle$ and $|-\alpha\rangle$. In

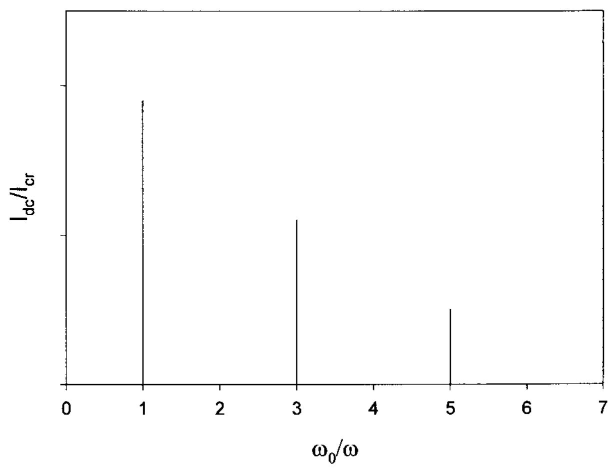

Fig. 2. Plot of $I_{\rm dc}/I_{\rm cr}$ *versus* scaled voltage bias $2eV_0/\hbar\omega$ for a Yurke-Stoler coherent state with $\alpha = |\alpha| = 1$. The values of the reported spikes are not to scale.

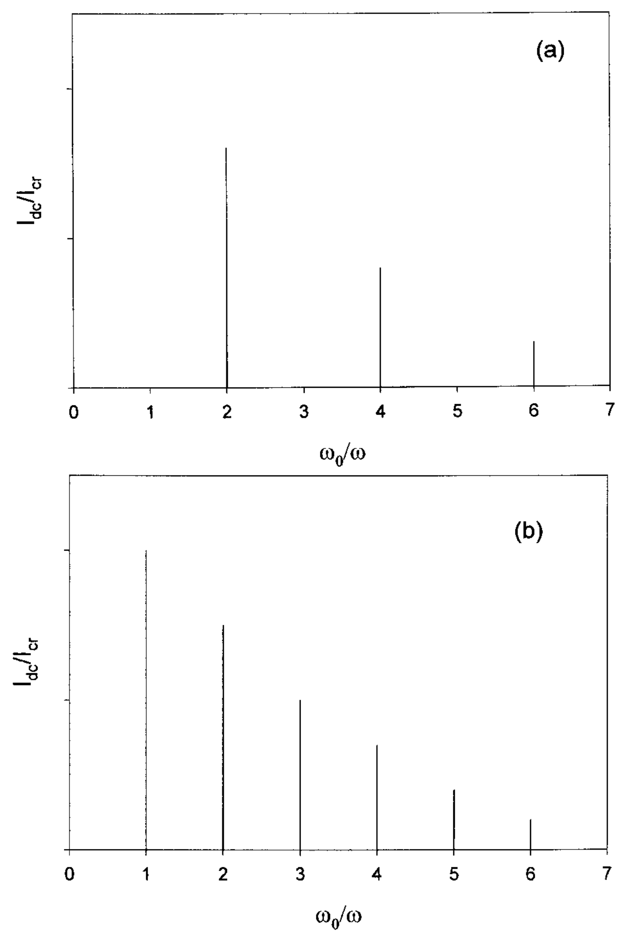

Fig. 3. Plot of $I_{\rm dc}/I_{\rm cr}$ *versus* $2eV_0/\hbar\omega$ for an even coherent state (a) and for an Yurke-Stoler coherent state (b), with α complex and $|\alpha| = 1$. The values of the reported spikes are not to scale.

particular it shows that the $I-V$ characteristic of the JJ does not exhibit any structure for even and odd coherent states ($\theta = 0$ and π) whereas, Shapiro steps are present when the Yurke and Stoler state ($\theta = \pi/2$) is considered. Figure 2 reports the first three dc-current spikes occurring at $\omega_0 = (2p+1)\omega$ with $p = 0, 1, 2$ relative to this last state.

Even more interesting results arise when α is complex. The position and the number of Shapiro steps depend both on θ_α and on θ. Some aspects concerning the behaviour of the mesoscopic Josephson junction in this case are reported in Figure 3.

In conclusion we wish to point out that an experimental confirmation of the results presented in this paper on the one hand would increase the applicative interest grown up over the last few years towards these superconducting devices and on the other hand would provide a good opportunity to witness the existence of macroscopic variables possessing quantum nature.

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