Observation of transition from escape dynamics to underdamped phase diffusion in a Josephson junction

J.M. Kivioja,[1,*] T.E. Nieminen,[1] J. Claudon,[2] O. Buisson,[2] F.W.J. Hekking,[3] and J.P. Pekola[1]

[1] *Low Temperature Laboratory, Helsinki University of Technology, POB 3500, FIN-02015 HUT, Finland*
[2] *Centre de Recherches sur les Très Basses Températures, laboratoire associé à l'Université Joseph Fourier, C.N.R.S., BP 166, 38042 Grenoble-cedex 9, France*
[3] *Laboratoire de Physique et Modélisation des Milieux Condensés, C.N.R.S. and Université Joseph Fourier, B.P. 166, 38042 Grenoble Cedex 9, France*

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We have investigated the dynamics of underdamped Josephson junctions. In addition to the usual crossover between macroscopic quantum tunnelling and thermally activated (TA) behaviour we observe in our samples with relatively small Josephson coupling E_J, for the first time, the transition from TA behaviour to underdamped phase diffusion. Above the crossover temperature the threshold for switching into the finite voltage state becomes extremely sharp. We propose a (T, E_J) phase-diagram with various regimes and show that for a proper description of it dissipation and level quantization in a metastable well are crucial.

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A hysteretic Josephson junction (JJ) switching from its metastable zero-voltage state into a stable state with a voltage of the order of twice the superconducting gap Δ has recently been used as a read-out device for superconducting quantum bit systems in many experiments [1]. Switching measurements also enable one to perform conventional large bandwidth current measurements, and recently there have been proposals to use hysteretic JJs as ammeters for studying phenomena like non-Gaussian noise [2]. Often it may be advantageous to reduce the critical current I_c of the detecting junction in order to increase the measurement sensivity. Yet the physics governing switching phenomena of small I_c junctions ultimately differs from those with larger I_c, and this sets a limit on how far one can reduce I_c still maintaining the useful features of the detector [3, 4].

The behavior of a JJ is determined by various parameters. Within the RCSJ (Resistively and Capacitively Shunted Junction) model these are the Josephson energy $E_J = \hbar I_c/2e$, the charging energy $E_C = e^2/2C_J$ (C_J is the junction capacitance), and a shunt resistance R responsible for dissipation. The dynamics of the junction are then characterized by the plasma frequency $\omega_p = \sqrt{8E_JE_C}/\hbar$ and the quality factor $Q = \omega_p RC$. As we will detail below, the behavior of the junction at a given temperature T can be classified according to the phase diagram of Fig. 1a. The overdamped case $Q < 1$ was studied in detail by Vion *et al.* [3], who uncovered the existence of a phase diffusion regime at finite T with the appearance of a small voltage, prior to switching to a voltage of the order of 2Δ. As far as the underdamped case $Q > 1$ is concerned, most experiments were done on large hysteretic junctions with relatively high I_c. Depending on temperature such junctions escape out of the metastable zero-voltage state either via macroscopic quantum tunnelling (MQT) or thermal activation (TA) processes, thereby switching directly to the finite-voltage state. In this Letter we focus on the regime $Q > 1$ with relatively small I_c. We show, theoretically and experimentally, that a regime exists where escape does not lead to a finite voltage state but rather to underdamped phase diffusion (UPD, the shaded region in Fig. 1).

According to the RCSJ model, the dynamics of a JJ biased with a current I is that of a phase particle (phase φ) with mass $m = \hbar^2/8E_C$, moving in a tilted cosine potential $U(\varphi) = -E_J\,(\cos\varphi + I/I_c\,\varphi)$ under a viscous force $(\frac{\hbar}{2e})^2\, 1/R\, \frac{d\varphi}{dt}$. The bias current renormalizes the plasma frequency, such that $\omega_p = \sqrt{\frac{d^2U/dq^2}{m}} = \frac{1}{\hbar}\sqrt{8E_JE_C}\; q_0^{1/2}$ with $q_0 = \sqrt{2(1 - I/I_c)}$. The cosine potential has wells where the phase particle can be localized; the phase then has constant average value, and the average voltage across the junction is zero. For non-zero I the quantum levels in the potential well are metastable and the particle can escape from a given well either via TA over, or MQT through the barrier. For large junctions $\Gamma_{TA} = \frac{\omega_p}{2\pi}\, e^{-\frac{\Delta U}{k_BT}}$ for the TA escape rate and $\Gamma_{MQT} = 12\sqrt{6\pi}\frac{\omega_p}{2\pi}\sqrt{\Delta U/\hbar\omega_p}e^{-\frac{36}{5}\frac{\Delta U}{\hbar\omega_p}}$ for the MQT rate [5]. In the cubic approximation the barrier height is $\Delta U = \frac{2}{3}\,E_J\, q_0^3$. Below the cross-over temperature $T_0 = \hbar\omega_p/2\pi k_B$ the dominant escape mechanism is MQT. The total escape rate is $\Gamma(I) \simeq \Gamma_{MQT}(I) + \Gamma_{TA}(I)$ and the total escape probability in the time interval $0 \le t \le \tau$ can be written as $P(I) = 1 - e^{-\int_0^\tau \Gamma(I)dt}$. If dissipation is weak, upon escape from the well the particle moves down the potential and phase is running freely, hence the voltage reaches a finite value (about $2\Delta_{BCS}^{Al} \approx 360\,\mu$V). However, if dissipation is strong enough, there is a finite probability that,

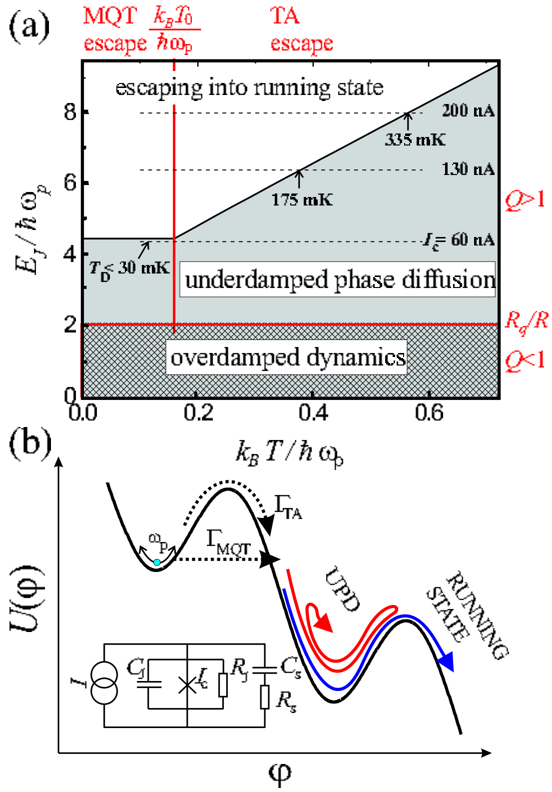

FIG. 1: (a) The various operation regimes of a Josephson junction with low E_J. For details, see text. (b) The cosine potential with dynamics *inside* the upper well and schematic dynamics *after* leaving the upper well. Inset: equivalent circuit of the junction with frequency dependent dissipation.

upon escape from the well, the particle is relocalized in the next well instead of running down the potential: the phase then has 2π-slips and diffusively moves from one metastable well to another. In this UPD regime the average voltage across the junction is much smaller than 2Δ [3].

A misconception still persists that phase diffusion is absent in a hysteretic junction, although it was pointed out over a decade ago that it can occur due to the dependence of dissipation on frequency ω [6]. Our experiment corresponds to the simplified equivalent circuit with frequency dependent dissipation as presented in the inset of Fig. 1(b). After switching to the running state, the dominant part of dissipation comes from small ω, governed by $R(\omega \sim 0)$, typically given by the large junction subgap resistance, of the order of 1 MΩ. In the phase diffusion regime the phase mainly oscillates in a well at the plasma frequency and thus the dissipation is characterized by $R(\omega_p)$, which is much smaller, typically of the order of vacuum impedance $Z_0 \approx 377\ \Omega$, since C_s acts as a short. In the following we will consider junctions that are underdamped even at ω_p, in contrast to [3].

The dissipated energy between neighboring potential maxima can be approximated by $E_D \approx 8E_J/Q$ and if the phase particle has energy less than E_D above the *next* barrier top, it simply diffuses to the next well. The maximum possible dissipated power due to phase diffusion can be written as $\frac{1}{2\pi}\frac{2eV}{\hbar}E_D$, where V is the average voltage across the junction. By equating this with the applied bias power I_mV, we find the maximum possible phase diffusion current

$$I_m = \frac{4}{\pi Q}I_c, \tag{1}$$

which is identical in form to the well known retrapping current formula, but now the value of Q is that at plasma frequency ω_p. For $I < I_m$, there is non-zero probability that phase relocalizes after escape. With decreasing I_c, dissipation starts to play a more significant role. First, ω_p and hence the quality factor Q are decreasing with decreasing E_J. On the other hand the escape rate is significant in the range of currents where the barrier height is comparable to the thermal energy. For large junctions, $E_J \gg k_BT$, this is the case when I is just slightly below I_c, but for small junctions this occurs even *without* tilting the cosine potential (I/I_c=0). At small values of I_c the dissipation *after* escape is thus also important, because the successive barrier tops are close in energy. The grey area in Fig. 1(a) presents the UPD regime, where escape does not necessarily lead to the transition into a running state. The minimum E_J^D, where such a transition after escape is still certain at temperature $T_D > T_0$ with current pulses of length τ,

$$E_J^D \approx \frac{3}{2}k_BT_D(1 - I_m/I_c)^{-3/2}\ln(\omega_{\rm p}\tau/2\pi), \tag{2}$$

determines the line separating TA and UPD regions in Fig. 1(a). For $T < T_0$, the escape rate equals the TA rate at T_0, thus E_J^D is independent of T, and given by Eq. (2) with T_0 replacing T_D. The UPD region in Fig. 1(a) corresponds to R_s= 500 Ω, C_J= 100 fF and τ = 100 μs.

We present experimental data of two samples, a dc-SQUID and a single JJ. The dc-SQUID consists of two wide superconducting planes connected by two short superconducting lines with tunnel junctions in the middle forming the dc-SQUID loop of area $20 \times 39\ (\mu\text{m})^2$ (see the inset in Fig. 2). The loop inductance was measured to be around 100 pH, small as compared to the calculated Josephson inductance ($L = \frac{\Phi_0}{2\pi I_c} = 400$ pH per junction). The dc-SQUID thus behaves almost like a single JJ, whose I_c can be tuned. The loop inductances were estimated from the measured resonant voltage determined by C_J and loop inductance [7]. The other measured sample was a single junction between long inductive biasing lines. The normal state resistances of the dc-SQUID and the single JJ were 1.3 kΩ and 0.41 kΩ yielding for I_c 199 nA and 630 nA, respectively. The geometrical capacitances of the samples were 100 fF and 130 fF. The measured samples were fabricated using standard electron beam lithography and aluminum metallization in a UHV evaporator. Both measured samples had strongly hysteretic IV-characteristics with retrapping currents well below 1 nA.

The experimental setup is presented in the inset in Fig. 2. Switching probabilities have been measured by applying a set of trapezoidal current pulses through the sample and by measuring the number of resulting voltage pulses. At the sample stage we used low pass RC-filters (surface mount components near the sample). In the measurements on a single junction we used surface mount capacitors ($C_s = 680$ pF), but in the dc-SQUID measurements we had π-filters in series with resistors , with $C_s \sim 5$ nF capacitance to ground. The resistors were $R_s = 500\,\Omega$ and $681\,\Omega$ in the measurement on a dc-SQUID and on a single junction, respectively. Bonding wires with inductance of order nH connect the sample to filters.

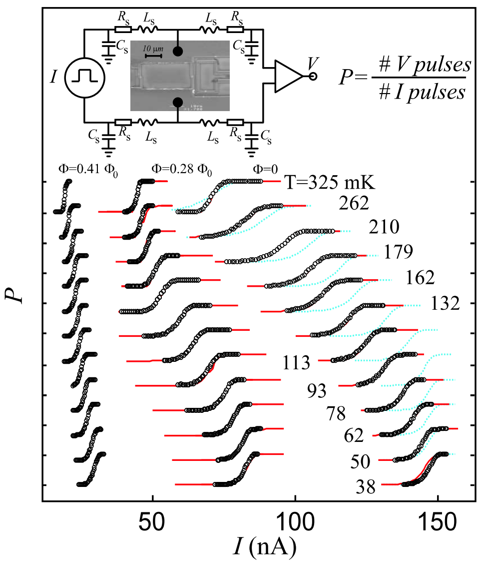

FIG. 2: Cumulative histograms of the dc-SQUID at different temperatures and at different magnetic fields. Curves are shifted for clarity and the vertical spacing between ticks corresponds to unity escape probability. Solid lines are from simulations described in the text; dotted blue lines show the results of basic MQT and TA models. Inset: scanning electron micrograph of the measured dc-SQUID and the experimental circuit.

In Fig. 2 we present the measured cumulative switching histograms (open circles) of the dc-SQUID at different fluxes Φ and temperatures with $\tau = 200\,\mu s$. At the lowest temperatures, the histograms can be well fitted by the MQT model, giving $I_c = 200$ nA, 128 nA and 55 nA, for $\Phi/\Phi_0 = 0$, 0.28 and 0.41 respectively. For $\Phi = 0$ we also present the simulated histograms based on the basic TA and MQT models without dissipation (dotted blue lines). In Fig. 3 we show the measured histogram position $I_{50\%}$ $[P(I_{50\%}) \equiv 0.5]$ and the width ΔI ($\equiv I_{90\%} - I_{10\%}$) for both samples. The dc-SQUID measurements were done both at negative and positive values of flux in order to ensure that the external flux had not changed. The position in Fig. 3(a) is normalized to the corresponding value of I_c at zero temperature. We also present the width and position results of the basic TA and MQT model simulations. At low T all the measured data are consistent with MQT results. On increasing T the parameters are constant up to the estimated cross-over temperature T_0. For $T > T_0$ the width is increasing and the position is moving down as TA model predicts. The qualitative agreement is good for most of the results up to the temperature T_D. At T_D, ΔI starts to decrease abruptly. Moreover, the position $I_{50\%}$ saturates at the same value $\simeq 0.35 I_c$. For the single JJ, the saturation occurs at $\simeq 0.3 I_c$. The dc-SQUID data measured at $\Phi = \pm 0.41\Phi_0$ are not following the standard theory even at low temperatures, since $T_D < 30$ mK in this case. If we assume a realistic shunt impedance $R(\omega_p) \simeq 500\,\Omega$ (the value of the surface mount resistors), we obtain $Q \approx 4$ ($I_c = 200$ nA and $C_J = 100$ fF), which yields $I_m \simeq 0.35 I_c$ through (1) like in the experiment. In the diagram of Fig. 1 we also present I_c of the dc-SQUID at fluxes 0, $\pm 0.28\Phi_0$ and

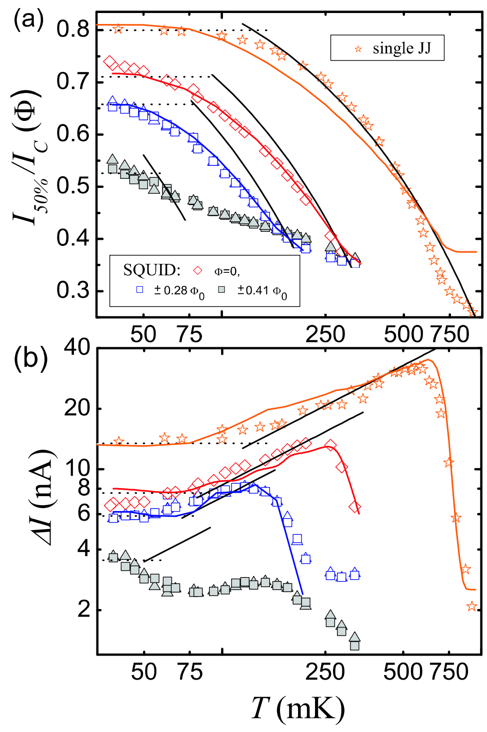

FIG. 3: (a) The positions ($I_{50\%}$) and (b) the widths (ΔI) of the histograms. Black solid and dotted lines are results (with known junction parameters) of TA and MQT model, respectively, ignoring dissipation. Blue, red and orange solid lines are the parameters of the simulated histograms based on the LO model discussed in the text. It assumes escape and possible relocalization events to be rare; thus it is not valid for data at $\Phi = \pm 0.41\Phi_0$, where the rate of phase diffusion events approaches the relaxation in the well.

$\pm 0.41\Phi_0$ by horizontal dashed lines. It can be seen that the intersections of the dashed lines and the boundary of the UPD regime are very close to the experimental values of T_D. The saturation of the histograms and their re-entrant steepness is thus a manifestation of the crossover from TA escape into UPD due to dissipation. In the case of a single JJ with $R(\omega_p) \simeq 680\,\Omega$ we obtain $Q \approx 13$ yielding $I_m \approx 0.1\, I_c$ and $T_D \approx 1.2$ K, instead of the measured 0.3 I_c and 650 mK. By taking $R(\omega_p) \simeq 230\,\Omega$ we obtain $Q = 4.4$, yielding $I_m \approx 0.3\, I_c$ and $T_D \approx 700$ mK instead, which is consistent with what we obtain from the position I_m/I_c.

Fig. 2 shows that the plain TA-MQT models cannot account for our observations. Except for the data at the lowest temperatures the width and the position of the measured histograms deviate from simulated ones (dotted line). Dissipation alone cannot explain the difference. The basic TA model yields $\Delta I \propto T^{2/3}$ [5] and it can be seen in Figs. 2 and 3 that the dc-SQUID has weaker temperature dependence even well below T_D. In the dc-SQUID there are just few energy levels in the well and thus the assumptions of continuous energy spectrum are not valid here [5]. The semiclassical model of Larkin and Ovchinnikov [8] takes into account the influence of the shape of the potential, in particular the fact that it is not harmonic. The total escape probability is calculated using $P_{esc}(\tau) = 1 - \sum_k \rho_k(\tau)$, where $\rho_k(\tau)$ is the probability of finding the particle in a state k after the current pulse of length τ. The kinetic equation of the phase particle can be written as $\frac{d\rho_k}{dt} = \sum_j (\gamma_{kj}\rho_j - \gamma_{jk}\rho_k) - \Gamma_k \rho_k$. We take into account only transitions γ_{jk} between neighboring levels and tunnelling out, Γ_k. The relaxation rates between levels j and $j-1$ are well approximated by $\gamma_{j-1,j} = j\omega_p/4Q$. We assume detailed balance and write $\gamma_{j,j-1}{=}e^{-\beta(E_j - E_{j-1})}\gamma_{j-1,j}$. The positions and the escape rates are calculated using the results in Ref. [8]. The final state $\overline{\rho} \equiv [\rho_1\ \rho_2\ \ldots\ \rho_k]$ is calculated by numerically using $\overline{\rho}(\tau) = \frac{1}{\tau}\int_0^\tau e^{\mathbf{A}(t)}\overline{\rho}(0)dt$, where $\mathbf{A}$ is the transition matrix. I is set to zero in the beginning, and $\overline{\rho}(0)$ is Boltzmann distributed.

The effect of the relocalizing dissipation can be taken into account also in the quantized energy level model. Using again $E_D = 8E_J/Q$, and the fact that the energy difference between the two successive maxima is $-2\pi E_J I/I_c$, the level energy E must satisfy

$$\Delta U - E < E_J\,(2\pi I/I_c - 8/Q) \tag{3}$$

to allow switching into the free running state. If (3) is violated, the corresponding tunnelling rate is set to zero [9]. Note that (3) gives the same threshold as (1) for $\Delta U = E$, but in Eq. (3) we assume that after tunnelling the starting point is not at the potential maximum. The solid red lines in Fig. 2 present results of simulations with quantized energy levels and dissipation for the data of the dc-SQUID at zero and $\pm 0.28\Phi_0$ fluxes. At $\pm 0.41\Phi_0$ I_c is so small that the escape probability is large even at zero bias (except at the lowest temperatures). This means that the phase is moving constantly rather than infrequently escaping from a localized state, and thus our model does not work anymore. The only fitting parameter was Q, and the fitted values were in a very reasonable range. At $\Phi = 0$ we find $Q \approx 6$ at the lowest temperature, and it decreases with increasing temperature up to 4 at 325 mK. At $\pm 0.28\Phi_0$ $Q \approx 3$ to 4, again decreasing with temperature. In Fig. 3 is presented the $I_{50\%}$ and ΔI parameters of the simulated histograms. We present also the simulated parameters for single junction ($Q \approx 4$) [10]. In the measurements on a single junction, the number of energy levels is large (≈ 20) and the level separation is smaller than the level width. Our model assumes well-separated levels and thus the agreement with a simple TA model is better for a single junction at temperatures $T < T_D$, especially in Fig. 3 (a), whereas re-entrant steepness of the histograms could not be explained with the basic TA model. The agreement between the simulation and the measurements on a DC-SQUID is excellent. The position and the width of the measured histograms coincide and, in particular, at higher temperatures the simulated histograms for both samples also start to get steeper again. At higher temperatures the upper energy levels, whose escape rate is significant with smaller potential tilting angles, are populated as well. The histograms thus peak at smaller currents and the condition (3) is not necessarily fulfilled anymore. What remains in the measured (and simulated) histograms is the escape from the states at the tail of the Boltzmann distribution above the dissipation barrier.

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* Electronic address: jkivioja@boojum.hut.fi

[1] D. Vion *et al.*, Science **296**, 886 (2002); I. Chiorescu *et al.*, Nature **431**, 159 (2004); S.Saito *et al.*, Phys. Rev. Lett. **93**, 037001 (2004); J. Claudon, F. Balestro, F. W. J. Hekking, and O. Buisson Phys. Rev. Lett. **93**, 187003 (2004)

[2] J. Tobiska and Yu. V. Nazarov, Phys. Rev. Lett. **93**, 106801 (2004); J.P. Pekola, Phys. Rev. Lett. **93**, 206601 (2004).

[3] D. Vion *et al.*, Phys. Rev. Lett. **77**, 3435 (1996); Joachim Sjöstrand *et al.*, cond-mat/0406510 (2004);

[4] A. Franz *et al.* , Phys. Rev. B **69**, 014506 (2004).

[5] U. Weiss, *Quantum Dissipative Systems*, (World Scientific, Signapore, 1999), 2nd ed.

[6] J.M. Martinis and R.L. Kautz, Phys. Rev. Lett. **63**, 1507 (1989); R.L. Kautz and J.M. Martinis, Phys. Rev. B **42**, 9903 (1990).

[7] H.S.J. van der Zant, D. Berman and T.P. Orlando, Phys.

Rev. B **49**, 12945 (1994).
[8] A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz **91**, 318 (1986); A. I. Larkin and Yu. N. Ovchinnikov, Sov. Phys. JETP **64**, 185 (1987); A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz **87**, 1842 (1984); A. I. Larkin and Yu. N. Ovchinnikov, Sov. Phys. JETP **60**, 1060 (1984).
[9] With the low Q values in the experiment, the phase relaxes in the next well in a time $\sim \omega_p^{-1} \sim 100$ ps, which is far shorter than the typical time interval between phase diffusion events with studied current bias values (10...100 μs).
[10] We have not taken into account in the analysis that the superconducting gap is reduced when T is approaching $T_c \approx 1.2$ K.